Quantum DeepONet: Neural operators accelerated by quantum computing

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Introduction

A **quantum neural network** is an innovative type of neural network that integrates quantum computing principles to enhance data processing and learning capabilities, potentially surpassing classical neural network performance.

A **deep operator network (DeepONet)** is a machine learning architecture approximating the mapping between the input function space $\mathcal{V}$ and output function space $\mathcal{U}$:

$$\mathcal{G} : \mathcal{V} \ni v \mapsto u \in \mathcal{U}.$$

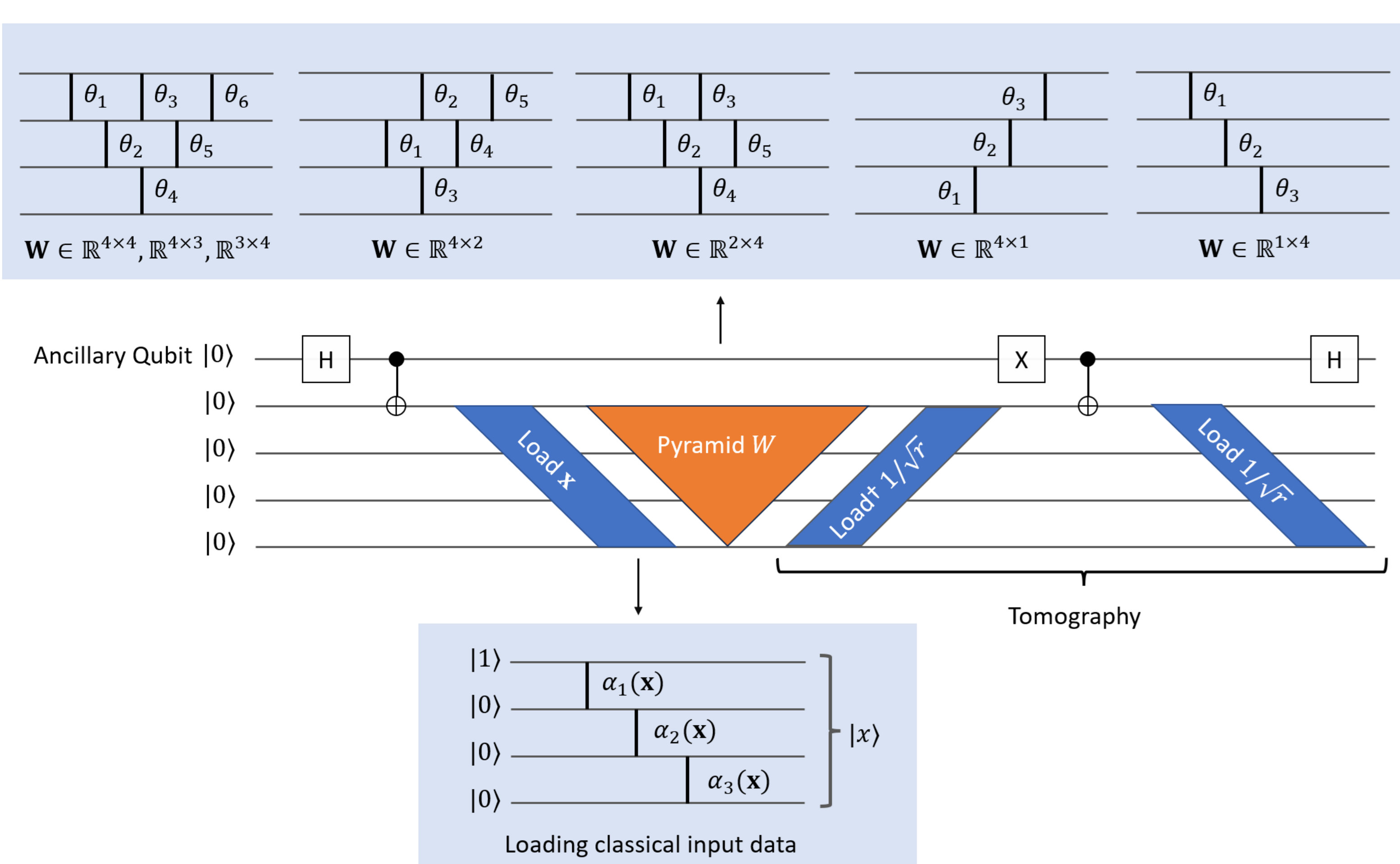
Once trained, DeepONet can be evaluated numerous times in evaluation, present an ideal application scenario for quantum neural networks designed for **accelerating the evaluation** process.

Quantum layers

We design an architecture for quantum DeepONet, utilizing reconfigurable beam splitter (RBS) gate

$$U_{RBS}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The neural network layer accelerated by quantum algorithm is referred as a **quantum layer**.



Methods summary

The workflow of our quantum method is divided into three distinct phases:

1. Training quantum DeepONet on classical computer;
2. Transferring of parameters to quantum layer;
3. Execution on quantum computer or simulator for evaluation.

Algorithm	Feedforward pass	Weight matrix update
Quantum DeepONet (evaluation)	$\mathcal{O}(n/\delta^2)$	–
Quantum DeepONet (train)	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$
Classical DeepONet	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$

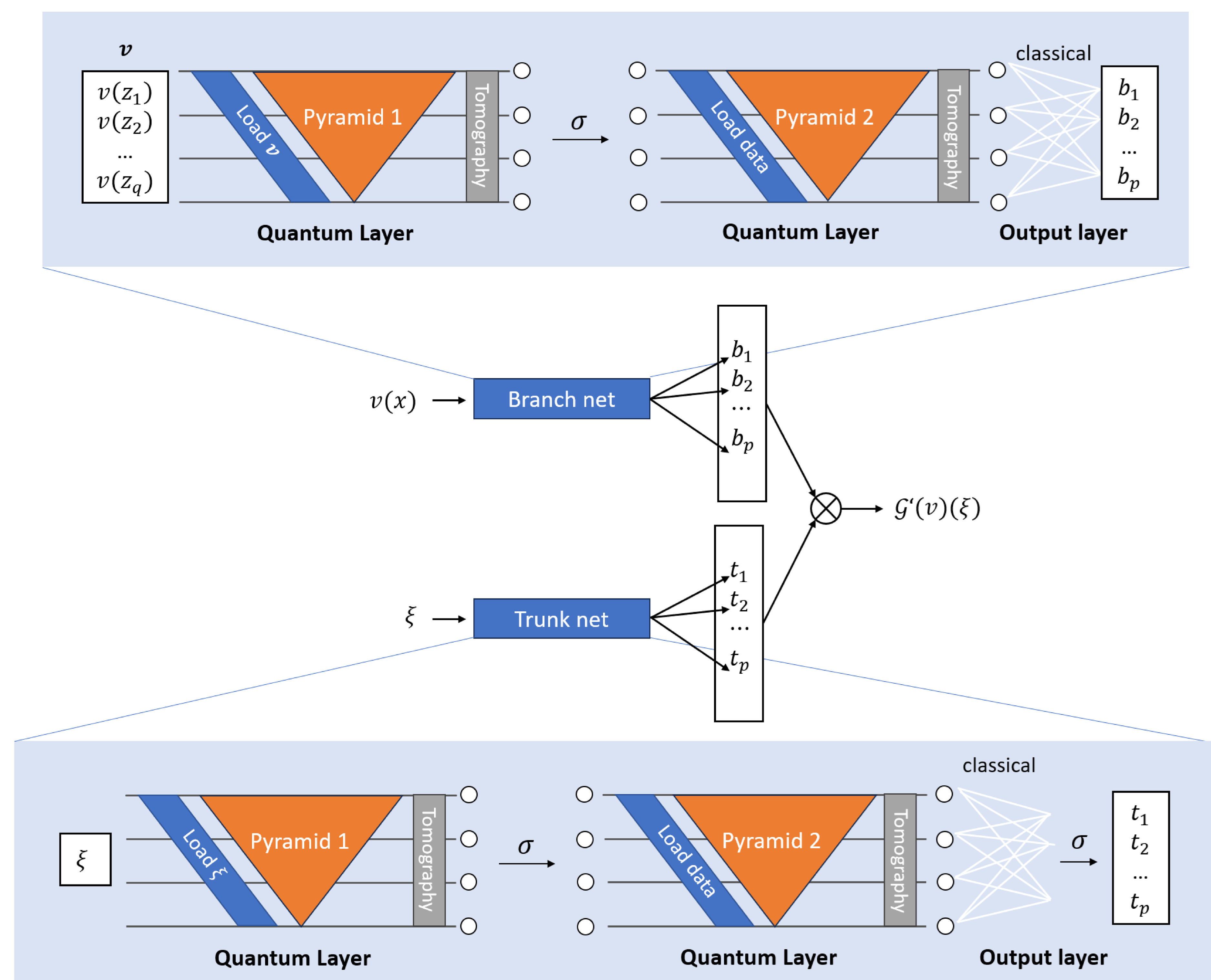
Table 1. Complexity. n is the input dimension and δ is threshold for the tomography error.

Quantum DeepONet

A DeepONet includes a branch net and trunk net, each with an equivalent number of output neuron, denoted by p. The final output of DeepONet is

$$\mathcal{G}'_\theta(v)(\xi) = \sum_{k=1}^{p} b_k(v)t_k(\xi) + b_0,$$

where $\mathcal{G}'$ denotes the learned approximation of operator $\mathcal{G}$, $b_0 \in \mathbb{R}$ is the bias, and θ is the trainable parameter of the network. In Quantum DeepONet, the classical neural network layers are replaced with quantum layers.



Ideal results

We first exclude quantum and statistical noise to assess the model's theoretical accuracy and performance.

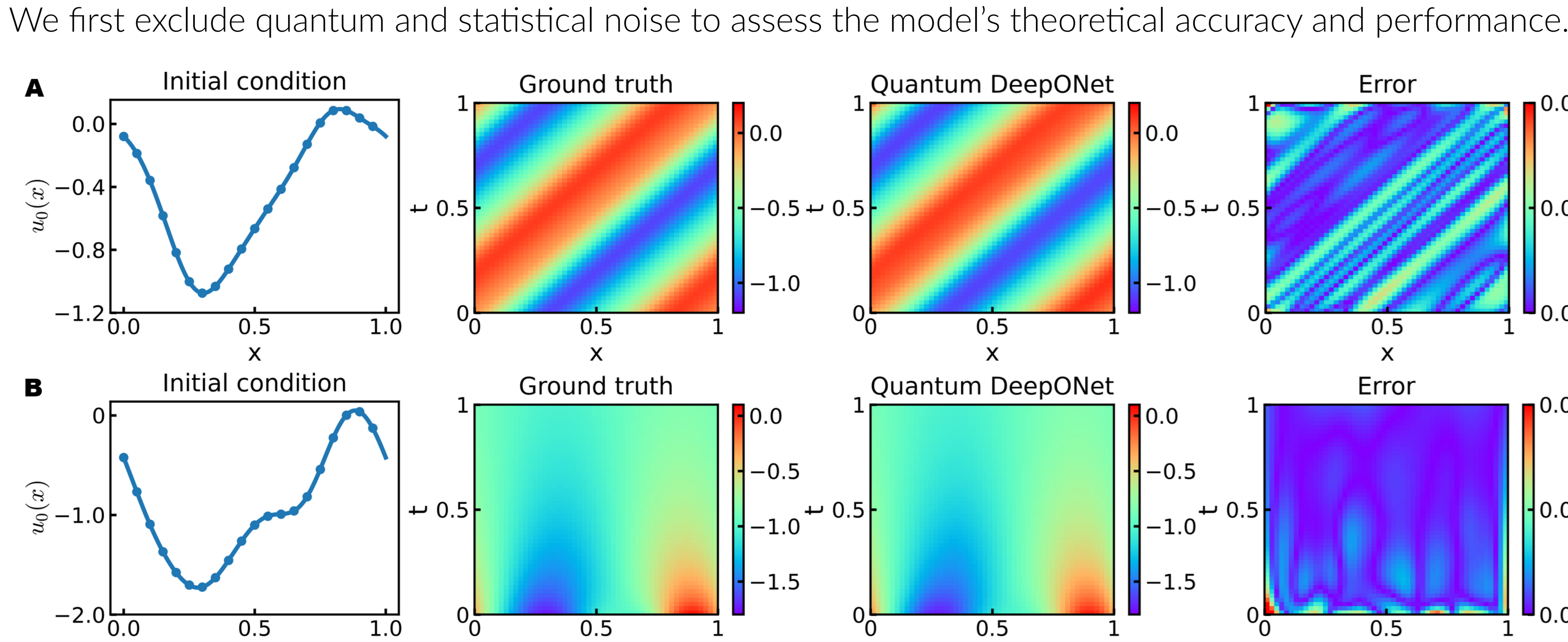


Figure 1. **Examples of quantum DeepONet prediction.** (**A**) Advection equation. (**B**) Burgers' equation.

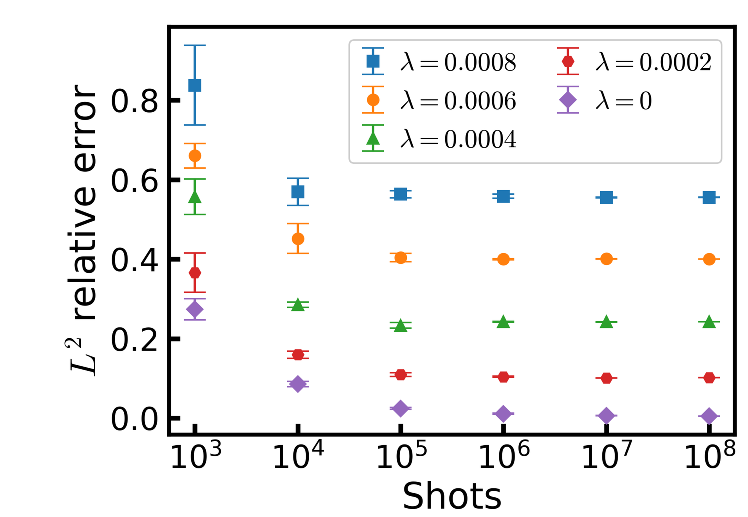
Quantum noise

Quantum noise is a major obstacle for the practicality of a quantum algorithm in the noisy intermediate-scale quantum (NISQ) era. We discuss the feasibility of our model under following types of noise:

- **Finite-sampling noise**, adjustable through the number of shots.
- **Depolarizing noise**, modifiable via the noise parameter λ, typically on the order of 10^{-4} in current quantum computers.

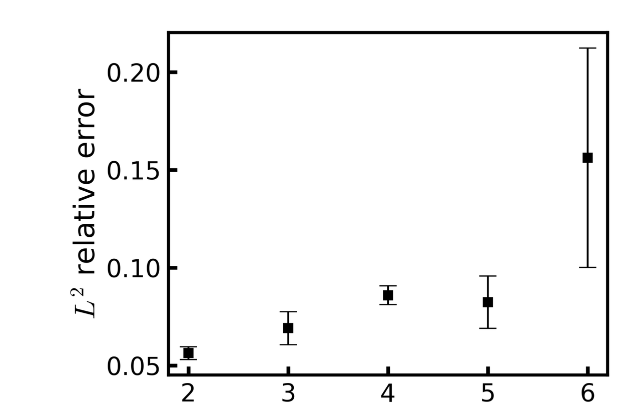
Noisy results

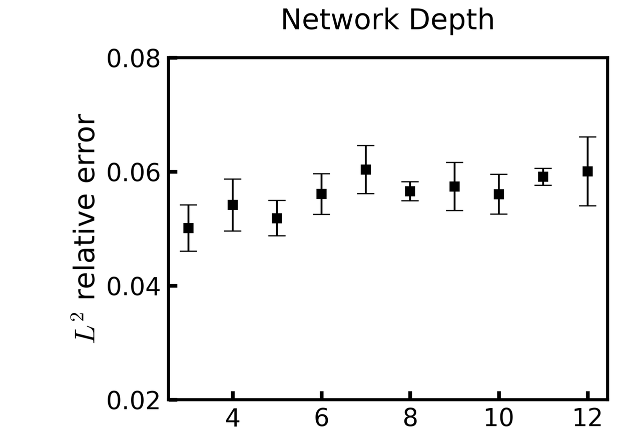
For the same neural network, error varies with changes in the number of shots and λ.



For the same noise level,

- Error increases exponentially with network depth.
- Error increases minimal with growing network width.





References

[1] Pengpeng Xiao, Muqing Zheng, Anran Jiao, Xiu Yang, and Lu Lu. Quantum deeponet: Neural operators accelerated by quantum computing. *arXiv preprint arXiv:2409.15683*, 2024.